

Game Theory

Part B

The Prisoner's Dilemma Game

One classic type of game is the *prisoner's dilemma game*. Prisoner's dilemma games are games in which each player has a dominant strategy; and when both players play the dominant strategy, the payoffs are smaller than if each player played the dominated strategy. The dilemma is how to avoid this bad outcome.

The basics of the prisoner's dilemma game are as follows: Two prisoners have the option to confess or not confess to a crime they committed. The prosecutor has only enough information to convict both criminals of a minor offense and is, therefore, relying on a confession. The minor offense carries one year in jail. The prisoners are questioned in different cells, without the ability to communicate. They are told that if one prisoner confesses while the other remains silent, the prisoner confessing will go free and the prisoner remaining silent will serve 20 years in jail. If both prisoners confess, both prisoners will serve three years in jail.

If a player goes free, the payoff is 0. If a player serves one year in jail, the payoff is -1 . If a player spends 20 years in jail, the payoff is -20 . Use these numbers in your payoff matrix. Note that the negative numbers come from losing years of freedom.

1. Determine the three basic elements of the game.
 - (A) The players: *Prisoner 1 and Prisoner 2*
 - (B) The strategies for each player: *Confess or Not Confess*
 - (C) The payoffs for each player: *If one confesses, he or she goes free, and the other gets 20 years in jail. If both confess, both get three years in jail. If neither confesses, both get one year in jail.*
2. Create a payoff matrix for the prisoner's dilemma game.

		Prisoner 2	
		Confess	Not Confess
Prisoner 1	Confess	$-3, -3$	$0, -20$
	Not Confess	$-20, 0$	$-1, -1$

3. Identify any dominant strategies. *Prisoner 1: Confess. Prisoner 2: Confess*
4. Identify any dominated strategies. *Prisoner 1: Not Confess. Prisoner 2: Not Confess*
5. Find the Nash Equilibrium. *Both Confess.*

Part C

Variation of the Prisoner's Dilemma Game

You are in a class with one other student. It is the end of the semester, and final exams are in a week. Your teacher has said the final exam will be graded so that anyone who scores the class average on the final exam will receive a "B" in the class. Anyone who scores above the average will receive an "A" in the class, and anyone who scores below the average will fail the class. You would certainly score higher on the exam than the other student. You and the other student have made an agreement not to take the final exam so that the class average is zero and you both receive "B" grades.

6. Determine the three basic elements of the game.

- (A) The players: *You and the Other Student*
- (B) The strategies for each player: *Take the Exam or Not Take the Exam*
- (C) The payoffs for each player: *If both players take the exam, you receive an A and the Other Student receives an F. If both players do not take the exam, both receive a B. If you take the exam and the Other Student does not take the exam, you receive an A and the Other Student receives an F. If the Other Student takes the exam and you do not take the exam, you receive an F and the Other Student receives an A.*

7. Create a payoff matrix for this game.

		Other Student	
		Take the Exam	Not Take the Exam
You	Take the Exam	A, F	A, F
	Not Take the Exam	F, A	B, B

8. What is your dominant strategy? (Underline the correct answer.)

Take the Exam or Not Take the Exam

9. Using a four-point scale ($A = 4$, $B = 3$, $C = 2$ and $D = 1$), which choice results in the highest class GPA? *Both players do not take the exam because $B + B = 3 + 3 = 6$ class GPA is higher than $A + F = 4 + 0 = 4$ class GPA.*

If you finished Parts B and C correctly, you will realize that when each player chooses his or her dominant strategy, the result is unattractive to the group.

The key to avoiding the prisoner's dilemma outcome of lower payoffs for both players is to find a way for players to credibly commit to playing a dominated strategy. Merely having both prisoners agree to Not Confess or both students to Not Take the Exam will not work. This results because it is always optimal for Prisoner 1 (or Prisoner 2) to still play the Confess strategy, and it is always optimal

for the better student to play the Take the Exam strategy. One possible way to have credible commitment in the prisoner's dilemma game would be to have both prisoners reveal another past crime they committed, thus ensuring that if they confess to this crime, the other prisoner will have additional information to punish the prisoner who cheats on an agreement to not confess.

One way to do this is to form a *cartel*. A cartel is a coalition of firms that coordinate their decisions to reach a more optimal solution for all members of the group by finding ways to credibly commit players to play their dominated strategies. Cartels, however, are not always successful in maintaining their agreements because there may be an incentive for a member to cheat on the cartel.

Part D

Questions

10. Is the Coke and Pepsi advertising game a prisoner's dilemma game? Explain why or why not.
Yes, it is optimal for both players to play their dominated strategies and be at the Don't Advertise/Don't Advertise corner, earning 100 each.

11. Interpret "standing at a concert" in terms of the prisoner's dilemma game. *If one person stands, he or she gets a better view of the concert. If the person in front of someone stands, then that person's best response is also to stand, or he or she will not be able to see the concert. However, if all people sat, then everyone would be able to see the concert and would not have to get tired standing.*

12. Explain at least one way the optimal outcome for players, which would be for all players to play the dominated strategy, can be reached in Question 11. What are the possible commitment problems?
The concert hall could require people at the concert to remain seated. However, this implies an external enforcer. If an external enforcer cannot be used, the group may collectively decide legal ways to punish those who stand. The punishment could range from throwing food at violators to physically assaulting them. The key is to make the commitment credible.

13. A rivalry exists between the U.S. jet producer Boeing and the European jet producer Airbus. Each government has the opportunity to subsidize its jet producer to give it a competitive edge in the global market. Using game theory, explain what you would expect to observe in practice.
Both countries would subsidize their producers. However, this costs money and lowers the price of jets for the rest of the world without either firm ultimately receiving a competitive advantage (the same outcome for both firms if there were no subsidies at all). This is another example of the prisoner's dilemma game.